The **Gradient Field** of a differentiable function f is the field of gradient vectors.

In 2D, the gradient field of f(x,y) is the set of vectors $\nabla f(x,y) = \left\langle f_x(x,y),\ \ f_y(x,y) \right\rangle$
In 3D, the gradient field of f(x,y,z) is the set of
vectors $\nabla f(x,y,z) = \left\langle f_x(x,y),\ \ f_y(x,y),\ \ f_z(x,y) \right\rangle$

Work & Flow along a path -- Path Integrals

Let $r(t) = \langle\ x(t),\ y(t),\ z(t)\ \rangle$ be a smooth path and T(t) = unit tangent vector of r(t).

Work along r(t)

If $F(x,y,z) = \langle\ M(x,y,z),\ N(x,y,z),\ P(x,y,z)\ \rangle$ is (a field of) force vectors,

then the Work done by the force F(x, y, z) along the curve r(t) from t=a to t=b is

$$W = \int_{t=a}^{t=b} F \bullet T \ \ ds$$

What this really means – different forms of the same integral:

$$W = \int_{t=a}^{t=b} F \bullet T \ \ ds$$

$$= \int_{t=a}^{t=b} F \bullet dr \qquad \text{since } T = \frac{r'}{\|r'\|} \quad and \quad ds = \|r'\|$$

$$= \int_{t=a}^{t=b} F \bullet \frac{dr}{dt}\ dt \qquad \text{because of the Chain Rule (usually most useful form)}$$

$$= \int_{t=a}^{t=b} \left(M\frac{dx}{dt} + N\frac{dy}{dt} + P\frac{dz}{dt} \right) dt \qquad where\ \ M = M(x(t),y(t),z(t))$$

$$= \int_{a}^{b} Mdx + Ndy + Pdz \qquad \text{the most common form (yuck!)}$$

Flow along r(t)

If F is a continuous velocity field, then the **flow** along r(t) from t=a to t=b is

$$Flow = \int_{t=a}^{t=b} F \bullet T \ \ ds \quad \text{(and it has the same alternate forms as the work integral)}$$