Course Number

Instructor

Date

Section 2.2 Polynomial Functions of Higher Degree

Objective: In this lesson you learned how to sketch and analyze graphs of polynomial functions.

Important Vocabulary Define each term or concept.

Continuous The graph of a polynomial function has no breaks, holes, or gaps.

Extrema The minimums and maximums of a function.

Relative minimum The least value of a function on an interval.

Relative maximum The greatest value of a function on an interval.

Repeated zero If $(x - a)^k$, $k > 1$ is a factor of a polynomial, then $x = a$ is a repeated zero.

Multiplicity The number of times a zero is repeated.

Intermediate Value Theorem Let a and b be real numbers such that $a < b$. If f is a polynomial function such that $f(a) \neq f(b)$, then, in the interval $[a, b]$, f takes on every value between $f(a)$ and $f(b)$.

I. Graphs of Polynomial Functions (Pages 147–148)

What you should learn
How to use transformations to sketch graphs of polynomial functions

Name two basic features of the graphs of polynomial functions.

1) continuous
2) smooth rounded turns

Will the graph of $g(x) = x^7$ look more like the graph of $f(x) = x^2$ or the graph of $f(x) = x^3$? Explain.
The graph will look more like that of $f(x) = x^3$ because the degree of both is odd.

II. The Leading Coefficient Test (Pages 149–150)

What you should learn
How to use the Leading Coefficient Test to determine the end behavior of graphs of polynomial functions

State the **Leading Coefficient Test.**
As x moves without bound to the left or to the right, the graph of the polynomial function $f(x) = a_nx^n + \ldots + a_1x + a_0$ eventually rises or falls in the following manner:

1. When n is odd:
   a. If the leading coefficient is positive, the graph falls to the left and rises to the right.
   b. If the leading coefficient is negative, the graph rises to the left and falls to the right.
2. When n is even:
   a. If the leading coefficient is positive, the graph rises to the left and right.
   b. If the leading coefficient is negative, the graph falls to the left and right.

Example 1: Describe the left and right behavior of the graph of $f(x) = 1 - 3x^2 - 4x^6$.

Because the degree is even and the leading coefficient is negative, the graph falls to the left and right.

III. Zeros of Polynomial Functions (Pages 150–154)

What you should learn
How to find and use zeros of polynomial functions to sketch their graphs

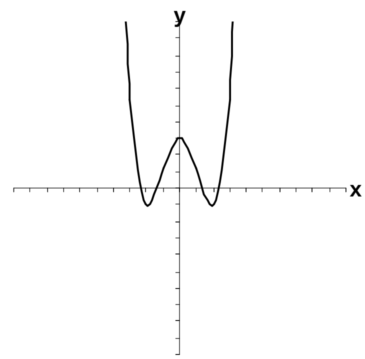
Let f be a polynomial function of degree n. The function f has at most ___ n ___ real zeros. The graph of f has at most ___ $n - 1$ ___ relative extrema.

Let f be a polynomial function and let a be a real number. List four equivalent statements about the real zeros of f.

1) $x = a$ is a zero of the function f
2) $x = a$ is a solution of the polynomial equation $f(x) = 0$
3) $(x - a)$ is a factor of the polynomial $f(x)$
4) $(a, 0)$ is an x-intercept of the graph of f

If a polynomial function f has a repeated zero $x = 3$ with multiplicity 4, the graph of f ___ touches ___ the x-axis at $x =$ ___ 3 ___. If f has a repeated zero $x = 4$ with multiplicity 3, the graph of f ___ crosses ___ the x-axis at $x =$ ___ 4 ___.

Example 2: Sketch the graph of $f(x) = \frac{1}{4}x^4 - 2x^2 + 3$.



IV. The Intermediate Value Theorem (Pages 154–155)

What you should learn
How to use the Intermediate Value Theorem to help locate zeros of polynomial functions

Interpret the meaning of the Intermediate Value Theorem.
If $(a, f(a))$ and $(b, f(b))$ are two points on the graph of a polynomial function f such that $f(a) \neq f(b)$, then for any number d between $f(a)$ and $f(b)$, there must be a number c between a and b such that $f(c) = d$.

Describe how the Intermediate Value Theorem can help in locating the real zeros of a polynomial function f.
If you can find a value $x = a$ at which f is positive and another value $x = b$ at which f is negative, you can conclude that f has at least one real zero between a and b.

Homework Assignment

Page(s)

Exercises