

No books or notes allowed. Show relevant work and put a box around your answer.

If you give an approximate numerical answer to a problem, use 3 places of accuracy after the decimal point.

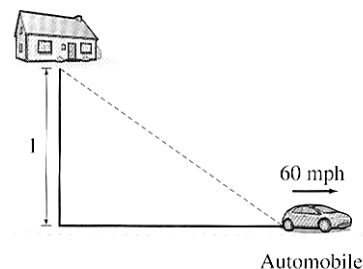
1 [18 points] Calculate the following derivatives. Show your work and box your answer.

(a) $\frac{d}{dx} [2x \tan(x^2 + 1)] =$

(b) $\frac{d}{dt} (\sin^4(7t + 1)) =$

(c) $\frac{d}{dx} (x^{\sin x}) =$

2 [16 points] A straight 1-mile road perpendicular to a highway leads to a house. A car travels on the highway at 60 miles per hour. How fast is the distance from the car to the house increasing when the car is 3 miles past the intersection of the road and highway?



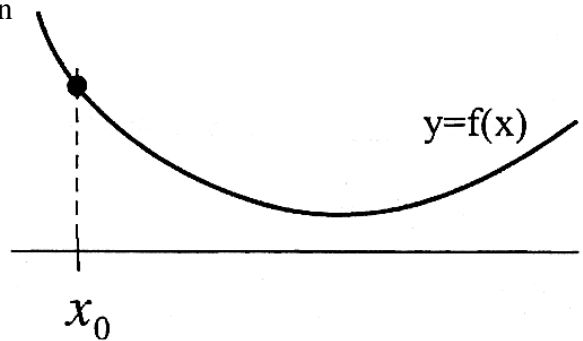
Answer: _____ (2 places after decimal) _____ (units)

3 [4 points] Find a function $h(x)$ with the derivative $h'(x) = \frac{7}{7x+4}$. $h(x) =$ _____

4 [13 points] Where does the formula $\frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$ come from? Give a neat and clear derivation of this formula by starting with the equation $y = \cos^{-1} x$ and proceeding as we did in class. (You may use the formula $\frac{d}{dx} \cos x = -\sin x$ without any explanation in your derivation.)

NEAT and CLEAR:

5 [6 points] The figure shows the graph of a function $f(x)$ and the location of x_0 . Find and LABEL the locations of x_1 and x_2 obtained by using Newton's Method.



6 [12 points] Figure 3 shows a parametric curve $(x(t), y(t))$. Assume that as t increases, the point $(x(t), y(t))$ moves without ever stopping.

(a) At $t = 4$, $\frac{dx}{dt}$ is **positive negative zero undefined**

(b) At $t = 4$, $\frac{dy}{dt}$ is **positive negative zero undefined**

(c) At $t = 2$, $\frac{dy}{dx}$ is **positive negative zero undefined**

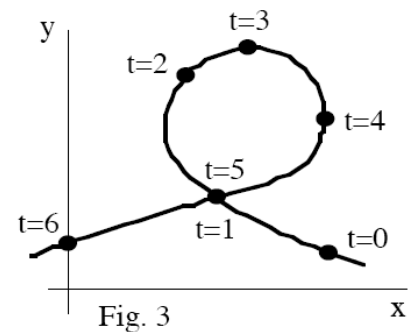


Fig. 3

7 [10 points] Suppose we wish to find an approximate solution to $\sin x = x - 1$ by using Newton's method. If we use an initial guess of $x_0 = 2$, find the values for the improved guesses x_1 and x_2 . Give 5 places after the decimal point for each. Show some work.

Answers: $x_1 =$ _____ $x_2 =$ _____
(5 places after the decimal for each)

8 [10 points] Each answer should be a number. Use the table of function values.

(a) At $x = 1$, $\frac{d}{dx} g(3x - 1) =$ _____

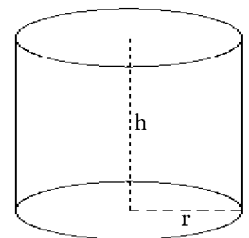
x	g(x)	g'(x)
0	3	-1
1	2	0
2	4	2
3	1	-3
4	1	3

x	h(x)	h'(x)
0	1	2
1	2	-2
2	2	3
3	2	-1
4	0	-2
5	1	3

(b) At $x = 1$, $\frac{d}{dx} h(g(x)) =$ _____

9 [12 points] A can has the shape of a cylinder, where the height of the cylinder is twice the radius of the circular top. If I measure the radius of the top and find it is 2 inches with a maximum possible error of 0.1 inch, what is the maximum possible error in the volume of the can when I compute it using the radius? (Note: the volume of such a cylinder is $V = 2\pi r^3$ where r is the radius on the circular top.)

(a) The differential $dV =$ _____
(in terms of r and dr)



(b) Use the differential to estimate the error in the volume when you compute it using $r = 2 \pm 0.1$ inches. You must use the differential, and give 4 places of accuracy after the decimal.

Answer: Estimate of error in the volume = _____ cubic inches.
(4 places after the decimal)