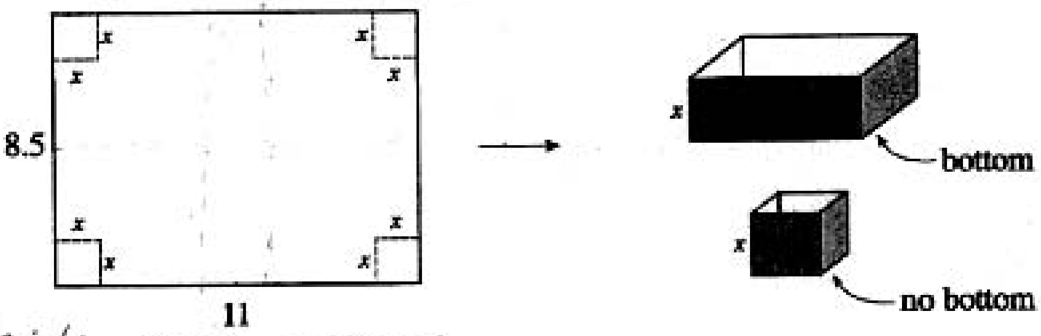
There is a traditional problem (your text has an example in Exercise 8) that goes like this: "We want to make an open-topped box from an 8.5 × 11 inch sheet of paper by cutting congruent squares from the corners and folding up the sides. What is the maximum possible volume of such a box?" What most people never think about is the fate of those four squares of paper. They don't have to be wasted. By taping them together, and putting the resultant structure on a desk, one can make a handsome pen-and-pencil holder, which will be a box with neither top nor bottom. (It will still hold pencils as long as it rests on the desk.)



endpoints $x=0$, $x=4.25$

1. What is the maximum possible combined volume of an open-topped box plus a handsome pen-and-pencil holder that can be made by cutting four squares from an 8.5 × 11 inch sheet of paper?

Volume of box: $V_b = x(8.5-2x)(11-2x)$

Volume of pencil holder: $V_p = x^3$

Total $= V_b + V_p = 5x^3 - 39x^2 + 93.5x$

critical #'s: $15x^2 - 78x + 93.5 = 0 \rightarrow x = \frac{78 \pm \sqrt{474}}{30} \approx 3.325$ or 1.874

$V|_{x=0} = 0$ $\quad V|_{x=3.325} \approx 63.51$ $\quad \boxed{V|_{x=4.25} \approx 76.76 \text{ in}^2}$

$V|_{x=1.874} \approx 71.16$

2. Describe the open-topped box that results from this maximal case. Intuitively, why do we get the result we do? Open top box is flat (zero volume)

Makes sense b/c we get volume without wasting paper on a bottom if we put as much as we can into pencil holder.

3. Repeat this problem for a 6 × 10 inch piece of paper.

$V = x^3 + x(6-2x)(10-2x) = 5x^3 - 32x^2 + 60x$

crit #'s: $15x^2 - 64x + 60 = 0 \rightarrow x \approx 1.39, 2.88$

endpoints $x = 0, 3$

$V|_{x=0} = 0$, $V|_{x=3} = 27$, $\boxed{V|_{x=1.39} = 35 \text{ in}^2}$, $V|_{x=2.88} = 26.8$